Topological derivation of black hole entropy by analogy with a chain polymer

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The generic crease set of an event horizon possesses anisotropic structure although most black holes are dynamically stable. This fact suggests that a generic almost spherical black hole has a very crumpled crease set on a microscopic scale although the crease set is similar to a pointwise crease set on a macroscopic scale. In the present article, we count the number of such microstates of an almost spherical black hole by analogy with an elastic chain polymer. This estimation of black hole entropy reproduces the well-known Bekenstein-Hawking entropy of a Schwarzschild black hole.

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I. INTRODUCTION: A TOPOLOGICAL VIEWPOINT OF EVENT HORIZONS

One of the most remarkable aspects of black hole entropy [1] is that it is not proportional to something like the volume of the black hole but to the area of its event horizon, while entropy is an extensive variable in statistical mechanics. Furthermore, if one tries to find the appropriate volumelike variable, even for a Schwarzschild event horizon, there is no such thing as a volume inside the horizon since it depends on the global solution one chooses which could even render the volume infinite, for a Cauchy surface.

From one point of view, this is interpreted as that the entropy is not of the black hole but of the event horizon that is the boundary of the black hole region. In this sense, some authors have derived black hole entropy by calculating the degrees of freedom only on the event horizon in various quantum theories, e.g., in quantum geometry [2] and by a technique in the setting of AdS/CFT correspondence [3], and others discuss the statistical meaning of the boundary in the context of entanglement [4]. Furthermore, the technique of the Euclidean path integral [5] is also on this basis, as the relevant contribution to black hole entropy comes from boundary integration at the event horizon.

On the other hand, the recently developed technique of the D-brane to derive the black hole entropy seems to be related to the whole of the black hole spacetime in itself [6], although it is not fully clear what is estimated by this derivation. In this sense, it may be valid to regard the black hole entropy as the entropy of the black hole region, after all.

If the black hole entropy is really of the black hole region, we will need a reason why it is proportional to the area of the event horizon. In this article, we try to estimate the entropy of the matter that has been absorbed into the black hole region during black hole formation, relating it to the topological structure of its event horizon. Then we can show that the black hole entropy is proportional to the area of the event horizon. Here we never relate the entropy directly to the area of the event horizon. The entropy concerns only the mass of the black hole. Moreover, this entropy can be regarded as the count of the ways to form the final black hole.

When we concentrate on the topological features of an event horizon, that can be reduced to the structure of the crease set of the event horizon [7]. On the crease set, two or more generators of the event horizon intersect and the event horizon is not smooth [8] (a rigorous definition will be given in the third section). Furthermore, catastrophe theory [9–11] tells that the generic crease set is composed not of the pointwise structure of a spherical black hole but of two-dimensional structures and their bifurcations. Taking an appropriate time slice, this two-dimensional crease set provides a toroidal event horizon. In this sense, the spherical topology of the event horizon is structurally unstable.

Here it should be noted that the above observations do not mean that a black hole and its crease set are always highly anisotropic. Since catastrophe theory suggests that the spherical topology changes under small perturbations on a corresponding microscopic scale, the degree of anisotropy would be very small in some cases. For example, when almost spherically symmetric matter collapses to an almost spherical black hole, on a microscopic scale its crease set will be highly distorted and bifurcated and its event horizon will have very complicated topology. On the contrary, on a macroscopic scale, the crease set can be treated approximately as a point and then the event horizon seems to have a spherical topology.

These aspects make us expect that the crease set will be endowed with microcanonical entropy. In the present article, we estimate the entropy of the crease set by analogy with a chain polymer, since the one-dimensional crease set possesses similar structure to a chain polymer (and the two-dimensional one will be similar in microcanonical statistics). Assuming that a multiply folded crease set forms zigzags like a chain polymer, we can estimate the microcanonical entropy of the crease set. Then we achieve the entropy of an almost spherical black hole that is coincident with the Bekenstein-Hawking entropy. Finally, we are going to interpret this entropy as the missing information on falling matter. In other words, the entropy counts the ways to form a final black hole.

In the next section, we recall the way to calculate the entropy of a chain polymer in a simple Ising model. The third section shows how one can estimate the entropy associated with the black hole from the viewpoint of its topologi-

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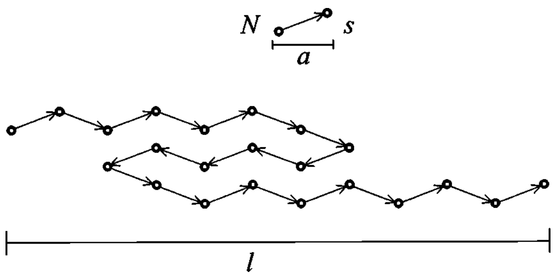

FIG. 1. A simple Ising model of the elasticity of a chain polymer is exemplified. A chain polymer is composed of N monomers with length a. Each monomer can be directed only to the left or the right. Although the maximum length of the chain should be Na, the highly folded chain has length $l \ll Na$.

cal structure. The final section is devoted to summary, discussions, and speculations.

II. ENTROPY OF CHAIN POLYMER

In this section, we recall the simple Ising model of elasticity of a chain polymer [12]. Suppose a large number N of monomers with a length a form a chain polymer with a total length Na. Furthermore, suppose this polymer is folded into an arbitrary length $l \ll Na$. To give a simple model of folding, we suppose that each element of the monomers can be directed only to the right or the left with equal probabilities as exemplified in Fig. 1.

In microcanonical statistics, the length l is a parameter describing the state of the system. The number of allowed configurations $W(l)$ is given by

$$W = \frac{2N!}{N_{\rightarrow}!N_{\leftarrow}!} = \frac{2N!}{\left(\frac{1}{2}N - l/a\right)!\left(\frac{1}{2}N + l/a\right)!}, \tag{1}$$

where $N_{\rightarrow}$ and $N_{\leftarrow}$ are the number of right- and left-directed elements, respectively. Then, using the Stirling formula $\log N! \simeq N \log N - N$, the entropy of this polymer becomes

$$\begin{aligned} S &= \log W \\ &\simeq N \log N - \left(\frac{1}{2}N - l/a\right)\log\left(\frac{1}{2}N - l/a\right) \\ &\quad - \left(\frac{1}{2}N + l/a\right)\log\left(\frac{1}{2}N + l/a\right). \end{aligned} \tag{2}$$

Under the assumption that the length l is much smaller than Na, this is approximated as

$$S(l) = N \log 2 - \frac{l^2}{2Na^2} + O(N\cdot(l/Na)^4), \tag{3}$$

$$= S(l=0) - \frac{l^2}{2Na^2} + O(N\cdot(l/Na)^4), \tag{4}$$

where we have used $\log(1+x) = x - x^2/2 + \cdots (x \sim l/Na \ll 1)$.

This gives a simple model of elasticity. Indeed, from the first law of thermodynamics $TdS = dU - fdl$ (T and U are the temperature and the internal energy), the elastic force f obeys the well known Hooke's law in leading order:

$$f = T\left(\frac{\partial S}{\partial l}\right)_U = -\frac{Tl}{Na^2}; \tag{5}$$

namely, the force is proportional to the temperature T. Actually, a rubber band contracts when it is warmed up, while an iron wire expands.

III. BLACK HOLE ENTROPY

Now we estimate the entropy associated with the crease set of an event horizon. Here we should give the definition of the crease set [8]. We consider a null vector field K on the event horizon that is tangent to the null geodesics generator λ of the event horizon. K is not affinely parametrized, but parametrized so as to be continuous even on the endpoint where the caustic of λ appears. Then the endpoints of λ are the zeros of K, which can become only past endpoints, since λ must reach infinity in the future direction. Of course, using an affine parametrization, K becomes ill defined at a subset of the set of the endpoints. We call such a subset the *crease set*. To be precise, we define the crease set by the set of the endpoints contained by two or more null generators of the event horizon. Thus the set of the endpoints consists of the crease set and endpoints contained by one null generator. The closure of the crease set contains the set of the endpoints of the event horizon generators, and the event horizon is undifferentiable there [7,8].

From Ref. [7], the spatial topology of an event horizon in a time slicing is determined only by the time slicing of the crease set. This implies that the crease set possesses all of the topological information of the black hole. In other words, an event horizon is completely determined once we know the crease set and all of the light rays starting from the crease set, since the event horizon should be their envelope. Hence we expect that the crease set will give all of the global information about the event horizon, while the light rays can be determined only by a local geometry. In this section, we try to estimate the entropy associated with that global information carried by the crease set of the event horizon.

In our point of view, the entropy of the crease set is brought by the missing information of falling bodies when they fall beyond the event horizon of a black hole. Since the crease set is the multiple point of the generator of the event horizon, its own structure will be changed provided that a falling body effects a congruence of the generators of the event horizon when the falling body crosses it. Then both the topology of the event horizon and the structure of the crease set reflect some information included in the configuration of matter outside the black hole. If we suppose that the topology of a black hole finally settles to a single spherical one after all the outside matter has fallen into the black hole, this information about the topology of the event horizon turns out to be absorbed into the black hole and translated into the information of the crease set. Therefore we expect that the missing part of this crease set information will correspond to the black hole entropy, and we try to estimate the entropy of the crease set.

To consider the degeneracy of the crease set, one might assign degrees of freedom to each Planckian scale segment of the crease set as the simplest model. The crease set, however, can be two, one, or zero dimensional. Each fundamental element becomes a Planckian area or length or vanishes, respectively. For example, the entropy of a one-dimensional crease set with a length L will intuitively be estimated as $S = \log W = \log(C^{L/l_{pl}})$, where C is the number of possible states for each fundamental element. This is not what we expected, since L could not be proportional to the area of the event horizon in the case of a one-dimensional crease set.

On the contrary, by analogy with a chain polymer, we will derive entropy of the crease set proportional to the area of the event horizon in the following. To determine the entropy, we count the logarithm of the microstate degeneracy. Although there may be various models of the microstate, in the present article we apply the following very simple Ising model, similar to the chain polymer.

First, we consider only the case of a one-dimensional crease set for simplicity since the case of a two-dimensional crease set will be different only by a factor in the entropy. On the other hand, it is concluded that the pointwise crease set is not generic from catastrophe theory [9–11]. This implies that, even for an almost spherically symmetric collapse of matter, the matter and spacetime are not rigorously spherically symmetric "in a microscopic scale" because of an anisotropic small perturbation. This will cause a highly folded crease set, which is confined within a very small region. Then it is not pointwise on a microscopic scale but on a macroscopic scale (see the bottom left of Fig. 2).

There are many ways to fold and confine the crease set. Considering that a number of ideal small fundamental elements of the crease set fold, this situation is very similar to the chain polymer discussed in the previous section (compare Fig. 1 and Fig. 2). Then we count the number of allowed configurations and estimate the entropy, by analogy with the chain polymer.

In the case of the chain polymer, the entropy S_{CP} is given by Eq. (4) and we think that the entropy of the crease set S_C is the same as S_{CP}:

$$S_C(l) = S_{CP}(l) = S_{CP}(0) - \frac{l^2}{2Na^2}, \tag{6}$$

where l is the length of the crease set. N and a are the number and length, respectively, of the ideal fundamental element.

In our discussion, the state with $l=0$ (the left branch of Fig. 2) is regarded as an almost spherically symmetric black hole, since this state is macroscopically similar to a spherical black hole with a zero-dimensional (pointwise) crease set. On the other hand, to make the black hole most anisotropic, the collapsing matter must be most tilted in a special direction. This configuration will not allow any degeneracy of the microstate. Nevertheless, the black hole is not allowed to take such an arbitrary tilted configuration; rather, it is natural

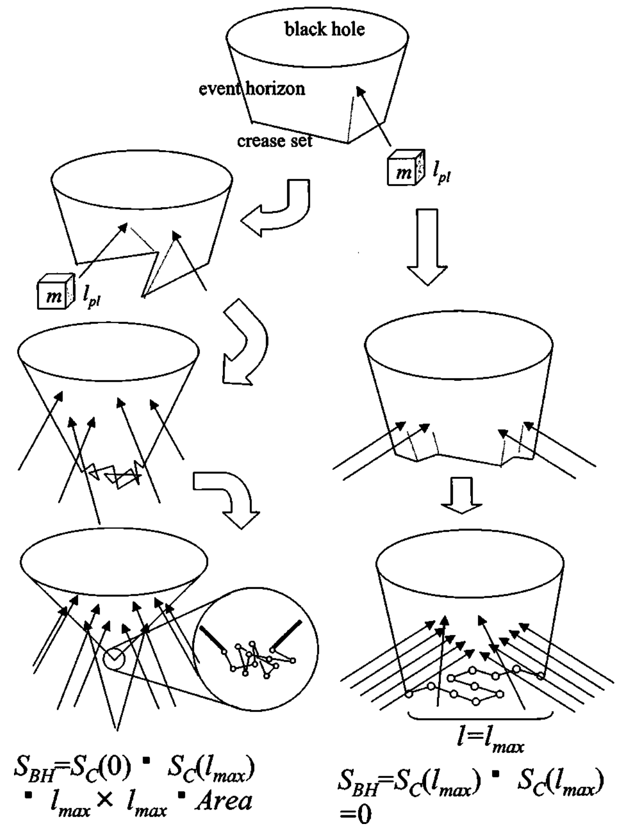

FIG. 2. Two types of black hole formation are illustrated. The left branch is an almost spherically symmetric collapse. A small volume element l_{pl}^3 with mass m affects a generator of the event horizon when it falls into the black hole. As illustrated on the top, this effect causes bending of the crease set of the event horizon. If many small bodies fall into the black hole from random directions at random time, the crease set is bent many times in various directions and confined in a small region. The resultant black hole seems almost spherically symmetric on a macroscopic scale. On the other hand, the right branch is an extremely anisotropic collapse. Since many of the falling bodies are ordered to be from a special direction, the bending effect will not make the crease set so small.

that there is an upper bound l_{max} for l since a black hole with infinitely large l seems to be unphysical. Then if we have an upper bound l_{max} (the right branch of Fig. 2), it is valid to regard $S(l_{max})$ as the zero point of the entropy of the black hole. Therefore the entropy of an almost spherical black hole is given by

$$S_{BH}(l=0) \equiv S_C(l=0) - S_C(l_{max}) = \frac{l_{max}^2}{2Na^2}. \tag{7}$$

We may expect that the upper bound l_{max} is about a final black hole mass M, since it is the only reasonable scale in gravitational dust collapse. Furthermore, the hoop conjecture [13] requires that the length of the crease set should be bounded by $2\pi M$ [14]. Hence we assume $l_{max} \simeq 2\pi M$. In addition, we assume $l_{max}/a \ll N$ in order to derive Eq. (4) in the previous section. The consistency and validity of this

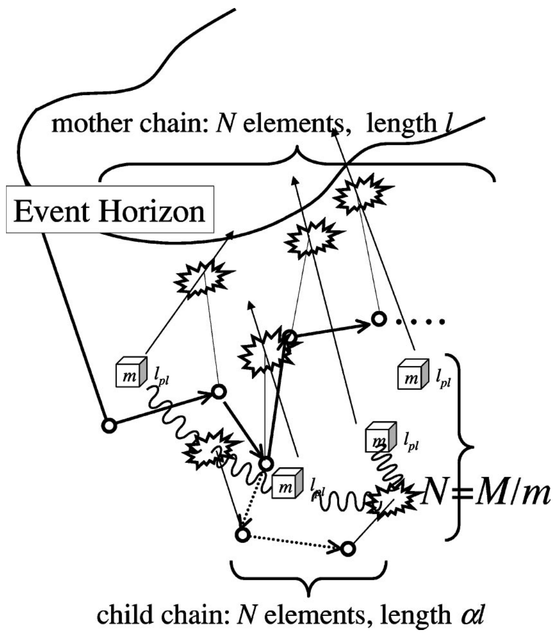

FIG. 3. The number of hinges of the mother chain is N, since the number of falling volume elements is $N=M$(total mass)/m(a mass of a volume). Bold (dotted) arrows are the segments of the crease set forming the mother (child) chain. A child chain develops from the hinge of the mother chain with probability β. Each small volume element of matter falls into the black hole along the arrows directed up. They cross the event horizon and affect its generators (narrow vertical line) at exploding symbols. On the other hand, three-point interactions affect the generators along the wavy lines. The child chain forms N hinges by these three-point interactions.

condition will be discussed later. Consequently, we observe that this entropy is proportional to the area of the event horizon $\mathcal{A}\propto M^2$.

By the way, Eq. (7) has an unfavorable factor a^2/N. Expecting that this entropy coincides with the Bekenstein-Hawking entropy, Na^2 should be on a scale of l_{pl}^2. This gives that a is $l_{pl}/\sqrt{N}\ll l_{pl}$ as N is a very large number. Since it is unreasonable to give a much smaller structure than the Planckian length to quantum spacetime, we cannot accept such a small a.

This problem of the scale of the small segment is resolved by considering the branches of the crease set. As pointed out in [9,11], there are possibilities that the crease set is branched at a hinge where the crease set can angle. We assume that a new branch (child chain) with a length αl (α is less than 1, since a child should be smaller than its mother by definition) occurs at some hinges with probability β (see also Fig. 3) and is composed of N elements; this will be justified later. The number of such a branch is given by the probability and the number of the mother's hinges as βN.

Moreover, there are also grandchildren and further descendants. Naively, the number of nth generation descendants might be considered to be $(\beta N)^n$ in geometrical progression. However, this is not realistic because we will require infinite volume to embed the whole family of the geometrical progression $\Sigma_i^\infty(\beta N)^i$. As suggested later, it seems that this divergence is related to the divergence of the many-body interaction. Then we require regularization for this divergence. Since the territory of a child and all its descendants would be limited around each of N hinges of the mother chain, we assume that the number of nth generation descendants is $\beta^n N$ rather than $(\beta N)^n$ as a regularization. By this assumption, the total length of the family becomes $N\Sigma_i^\infty\alpha^i\beta^i l$ and will converge so as to be embedded, since the nth generation descendant has length $\alpha^n l$. Then the total number of degenerate microstates is given by

$$W_{tot}=W(l)\cdot W(\alpha l)^{N\beta}\cdot W(\alpha^2 l)^{N\beta^2}\cdots, \tag{8}$$

where the nth factor is the contribution of all $(n-1)$th generation descendants. The entropy of the crease set (7) is changed by a factor and now we are not worried about the factor $1/N$ any more: the total entropy is

$$\begin{aligned}S_{Ctot}&=\log W_{tot}\\&=S_0-\frac{l^2}{2Na^2}-\frac{(\alpha l)^2}{2Na^2}N\beta-\frac{(\alpha^2 l)^2}{2Na^2}N\beta^2-\cdots+O(l^4)\\&=S_0-\frac{l^2}{2a^2}\left(\frac{1}{N}+\alpha^2\beta+\alpha^4\beta^2+\cdots+\alpha^{2n}\beta^n+\cdots\right)\\&\quad+O(l^4)\\&\sim S_0-\frac{\alpha^2\beta}{1-\alpha^2\beta}\frac{l^2}{2a^2},\end{aligned}$$

$$S_{BH}=\frac{\alpha^2\beta}{1-\alpha^2\beta}\frac{l_{max}^2}{2a^2},$$

where S_0 is the sum of all l-independent terms. On the third line, it is supposed that N is sufficiently large. Although the infinite sum might have any cutoff, it would change the result only by a numerical factor of order of magnitude 1.

If we rigorously require $S=\mathcal{A}/4l_{pl}^2=\pi M^2/l_{pl}^2$, the relation

$$\frac{4\pi^2M^2}{2l_{pl}^2}\frac{\alpha^2\beta}{1-\alpha^2\beta}=\frac{\pi M^2}{l_{pl}^2},$$

$$\frac{\alpha^2\beta}{1-\alpha^2\beta}=\frac{1}{2\pi},$$

will determine $\alpha^2\beta$, since the hoop conjecture says $l_{max}\sim 2\pi M$ and a should naturally be l_{pl}.

Now we must discuss the case of a nonchainlike crease set. Indeed, Refs. [10,9,11,15] tell us that it is important to consider a crease set with two dimensions. The discussion of a two-dimensional crease set can be proceeded with as follows. Intuitively, the two-dimensional crease set has two independent degrees of freedom to fold. This will make the state counting the square of that of a one-dimensional crease set and its entropy twice as large. For further accurate estimation, it might be valid to discuss using the theory of random surfaces. Similarly, in the case of the random surface, the regular term of its entropy around $l=0$ is also proportional to l^2 [16]. Thus the elastic force is always proportional to the amount of its deformation [see Eq. (5)] independently of its form, size, and dimensions. This is consistent with the general Hooke's law, i.e., a stress tensor is proportional to a distortion tensor. This consistency makes us convinced that our estimation is valid independently of the form, size, and dimensions.

So we summary the estimation as

$$S_{BH}=F(n)G(\alpha^2\beta)\frac{\mathcal{A}}{4l_{pl}^2}, \tag{9}$$

where $F(n)$ and $G(\alpha^2\beta)$ are numerical factors of order of magnitude 1, determined by the dimensions and branching of the crease set, respectively.

Finally, we discuss the assumptions we have made above. Here we should discuss the meaning of N and the validity of the assumptions about the size of it. In the present estimation we have supposed that the number of the mother's elements N is a fixed large number, and $0<l/a<l_{max}/a$ is much less than N.

One may be doubtful that these assumptions are consistent with the physical situation. To make this point clear, we consider the relation between N and the falling bodies as follows and illustrated in Fig. 2. We consider an ideal process in which some small elements with a volume l_{pl}^3 and a mass m fall into a black hole. The top of Fig. 2 illustrates that a falling body gravitationally deforms the generator of the event horizon, and consequently the crease set will form a hinge and be angled there. Here we note that the formation of the hinge occurs before the falling of the body in the sense of the usual spatial time slicing. Since the event horizon, however, is defined as the boundary of a past set, the mass of the falling body affects a past part of the event horizon along its null generators.

If many bodies randomly fall into the black hole (see the left branch of Fig. 2), the crease set will be repeatedly angled in various directions and finally confined into a small region. Therefore we guess that almost spherical collapse can occur through such a random falling process of a large number of small bodies. On the other hand, if the small bodies are not random but ordered to be anisotropic in a special direction (the right branch of Fig. 2), the crease set is more spread and an anisotropic black hole is formed. Hence the entropy of the crease set is related to the randomness of the falling bodies. To determine N, it is valid to relate the number of hinges of the crease set and the falling ideal volume elements with a volume l_{pl}^3, into which the collapsing matter can be decomposed.

Simply, we regard the number of collapsing ideal volume elements as the number of hinges of the mother chain N. A consistent interpretation of the child and descendant chain is the following (and see Fig. 3). When a falling body crosses event horizon generators, the mother chain is angled by the falling body directly. A child chain occurs (dotted arrows) with probability β. In addition, this child chain is also affected by another falling volume element through a three-point interaction (among one event horizon generator, one body making the child chain, and another body) since gravitation is a long range force. Therefore we consider that the hinges of the child chain are formed by this three-point interaction. It is well known that such many-body interactions diverge and need regularization. Here we think that this regularization corresponds to the assumption that the number of nth generation descendants is $\beta^n N$ rather than $(\beta N)^n$. Then the hinges of the child chain are assigned the two falling volume elements (indicated by wavy lines in Fig. 3); one of them made the child chain. Hence the child chain possesses N hinges. Similarly, an nth generation descendant chain also forms about N hinges under the influence of $(n+1)$ different falling volume elements. These pictures give an explanation for the formation and number of hinges of the descendant chain.

Now we consider the number of elements $N\sim M/m$, inherited from the number of falling volume elements. The mass m of the volume element of l_{pl}^3 should be much smaller than the Planckian mass so that it will not be a black hole but ordinary matter. Then we have the following inequalities:

$$N\sim\frac{M}{m}\gg\frac{M}{l_{pl}}\sim\frac{l_{max}}{l_{pl}=a}>\frac{l}{a}. \tag{10}$$

Therefore we have confirmed that all parameters are in a realistic range and the assumptions are consistent.

The picture illustrated in Fig. 2 might be something kinematical while the process we think of is dynamical. In other words, the picture gives the interpretation that this black hole entropy counts the logarithm of the number of ways to form an almost spherical black hole.

IV. SUMMARY, DISCUSSIONS, AND SPECULATIONS

In the present article, we argued that the Bekenstein-Hawking entropy of the Schwarzschild black hole can be derived independently of the area of the event horizon as the entropy of its crease set. This gives an interpretation to the black hole entropy, i.e., it measures the missing topological (global) information of the collapsing matter corresponding to the configuration of the falling volume elements in spacetime.

We considered only the Schwarzschild black hole as the final state of gravitational collapse. One may feel that it is important to extend the result to a rotating black hole. At present, however, we cannot imagine what shape of an event horizon is appropriate to compare with the Kerr black hole, as the zero of entropy. The discussion of the chain polymer should also be changed. To discuss these problems, we should relate the angular momentum to any character of the crease set, as the mass of black hole has been related to the maximum length l_{max} of the crease set.

Moreover, we would like to comment on the origin of the entropy estimated in the present article. As discussed at the end of the previous section, the entropy is related to the falling bodies. To be concrete, the entropy measures the disorder of the position and velocity of the falling bodies. Of course, this is not all the information that falling bodies carry. In other words, the black hole entropy could be directly related to only the entropy of this disorder. The black hole entropy is the logarithm of the number of possible configurations of falling matter to form a final Schwarzschild black hole, if we decompose the falling matter into ideal small volume elements l_{pl}^3 with mass $m \ll m_{pl}$ and omit the process where tilted black holes settle to a Schwarzschild black hole by radiating gravitational waves.

Finally, we estimate the upper bound of a thermal elastic force of the crease set. Substituting the Hawking temperature $T_H \sim 1/M$ into $f = -T\partial S/\partial l (l = l_{max})$, we observe that $f \sim 1/a^2$ is independent of M. Although its realistic meaning is not clear, this aspect coincides with the fact that the failure of Hooke's law occurs independently of the scale or form of the elastic body. Here we speculate that this coincidence implies the validity of the present discussions (in particular, the assumption $l_{max} \sim 2\pi M$ from the hoop conjecture). The mechanism of the failure of black hole formation or naked singularity formation, which is the basis of the hoop conjecture, might be realized by analogy with the existence of such an elastic limit.

As the reader has noticed, the present estimation does not work in different spacetime dimensions. This is because of the absence of the hoop conjecture in other spacetime dimensions. In turn, that fact might lead to new conjectures in other spacetime dimensions if we require that this estimation reproduce the Bekenstein-Hawking entropy also in other spacetime dimensions.

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